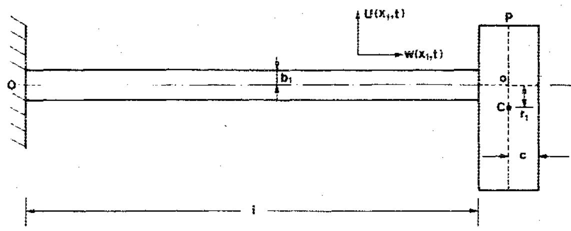

Fig. 1 Coordinate system and dimensions.

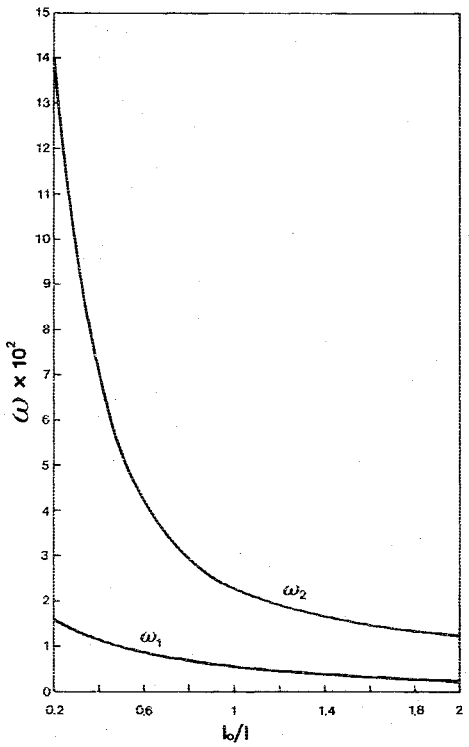

Fig. 2 Variation of ω_1, ω_2 with ℓ_0/ℓ for $\rho_0/\rho=2.945$, $b_1=0.1$ cm, $c=0.2$ cm, and $\ell=5$ cm.

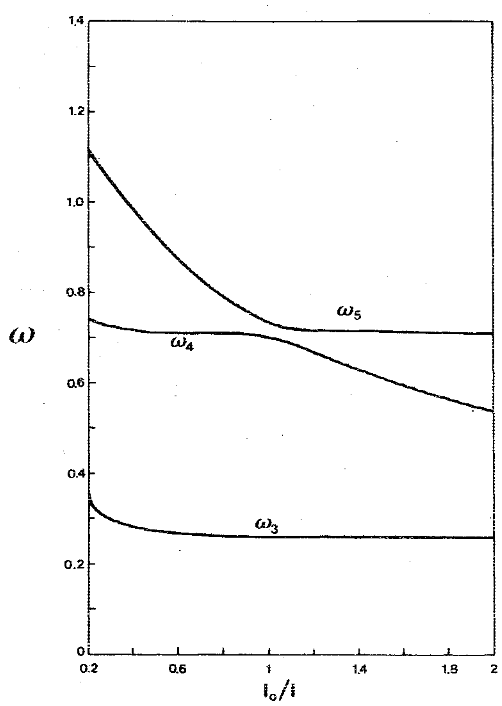

Fig. 3 Variation of ω_3, ω_4, ω_5 with ℓ_0/ℓ for $\rho_0/\rho=2.945$, $b_1=0.1$ cm, $c=0.2$ cm, and $\ell=5$ cm.

Equation (12) has been solved numerically for the first five natural frequencies for the numerical values of the parameters given earlier and over the interval $0.2\leq\ell_0/\ell\leq 2.0$. The results are shown in Figs. 2 and 3. The variations of ω_1 and ω_2 with ℓ_0/ℓ are plotted in Fig. 2, from which it is evident that both frequencies decrease monotonically as ℓ_0/ℓ increases. The value of the second frequency initially decreases quite rapidly, but for $\ell_0/\ell>1$ its rate of decrease is considerably diminished. From Fig. 3, it is evident that the dimensionless frequencies ω_3, ω_4, and ω_5 also decrease monotonically as ℓ_0/ℓ increases. Initially, ω_3 decreases slightly and thereafter remains virtually constant. On the interval $0.2\leq\ell_0/\ell\leq 1.0$, ω_4 decreases very slowly, whereas ω_5 decreases noticeably and almost linearly. Near $\ell_0/\ell=1$, the values of ω_4 and ω_5 differ by about 4%, but for $1<\ell_0/\ell<2$, the roles are reversed, with ω_5 decreasing only a very small amount and ω_4 decreasing rather sharply in an almost linear manner.

Acknowledgment

The author is indebted to Anik Bossuat for her capable assistance in performing the numerical computations.

References

[1] Bhat, R. and Kulkarni, M. A., "Natural Frequencies of a Cantilever with a Slender Tip Mass," *AIAA Journal*, Vol. 14, 1976, pp. 536-537.

[2] Bhat, B. R. and Wagner, H., "Natural Frequencies of a uniform Cantilever with a Tip Mass Slender in the Axial Direction," *Journal of Sound and Vibration*, Vol. 45, pp. 304-307.

[3] Anderson, G. L., "Natural Frequencies of Two Cantilevers Joined by a Rigid Connector at Their Free Ends," to appear in the *Journal of Sound and Vibration*.

Entrainment Characteristics of Unsteady Subsonic Jets

M. F. Platzer*
U.S. Naval Postgraduate School, Monterey, Calif.
and
J. M. Simmons† and K. Bremhorst‡
University of Queensland, Brisbane, Australia

Introduction

THE entrainment mechanism in turbulent jets has been a subject of considerable basic and applied interest for many years. Recently, this problem has received increased attention because of the need to develop compact, yet highly efficient thrust augmenting ejectors for VSTOL applications.[1] Several new techniques have been introduced or proposed to increase the jet entrainment, e.g., hypermixing,[2] swirling,[3] acoustic interaction,[4] and unsteady jet techniques.[5] It is the objective of this paper to present recent results on the entrainment characteristics of two types of unsteady jet flows, i.e., oscillating jets with time-varying jet deflection and pulsating jets with time-varying mass flow.

Received July 11, 1977; revision received Oct. 31, 1977.

Index categories: Nonsteady Aerodynamics; Nozzle and Channel Flow; Jets, Wakes, and Viscid-Inviscid Flow Interactions.

*Professor, Department of Aeronautics. Associate Fellow AIAA.

†Senior Lecturer, Department of Mechanical Engineering. Member AIAA.

‡Senior Lecturer, Department of Mechanical Engineering.

The use of oscillating jets for enhanced flow entrainment was first advocated by Viets[5] who also developed a rather ingenious fluidic jet actuation device. Other oscillating jet studies are cited in Ref. 6, but they do not contain entrainment measurements.

The favorable effect of pulsating jets on flow entrainment seems to have been first recognized during the development of the pulse jet engine.[7-9] Lockwood[7] also noted the generation of ring vortices due to pulsating flow, an effect later verified more clearly by Curtet and Girard.[10] Further pulsating jet studies are those of Johnson and Yang,[11] Didelle et al.,[12] Binder and Favre-Marinet,[13] Crow and Champagne,[14] and, very recently, Bremhorst and Harch.[15] The following section is a report of three different experiments which were conducted to assess the effectiveness of jet unsteadiness in enhancing flow entrainment.

Experiments

Definition of the edge of a turbulent jet raises subtleties which are discussed by Crow and Champagne[14] in terms of the turbulent (or inner rotational) region and the induced potential flow (or potential tails). In the experiments mean volumetric flow rates $Q(x)$ in the turbulent region of unsteady subsonic jets were determined at a number of distances x from the nozzle by integration of mean jet velocity distributions. A constant temperature hot-wire anemometer was used in all cases, and the mean of its linearized output was assumed to be proportional to the mean velocity in the direction of the center line of the nozzle. Errors arising from estimation of the edge of the turbulent region and from the influence of high ratios of rms to mean velocities near the edge of the jet are regarded as tolerable in this investigation.

Various measures of entrainment are defined in the literature. Here, entrainment is defined as $[Q(x)-Q_E]/Q_E$, where Q_E is the mean volumetric flow rate at the nozzle exit. Clearly, the entrainment differs by unity from the dimensionless local flow rates $Q(x)/Q_E$ which are presented in this paper.

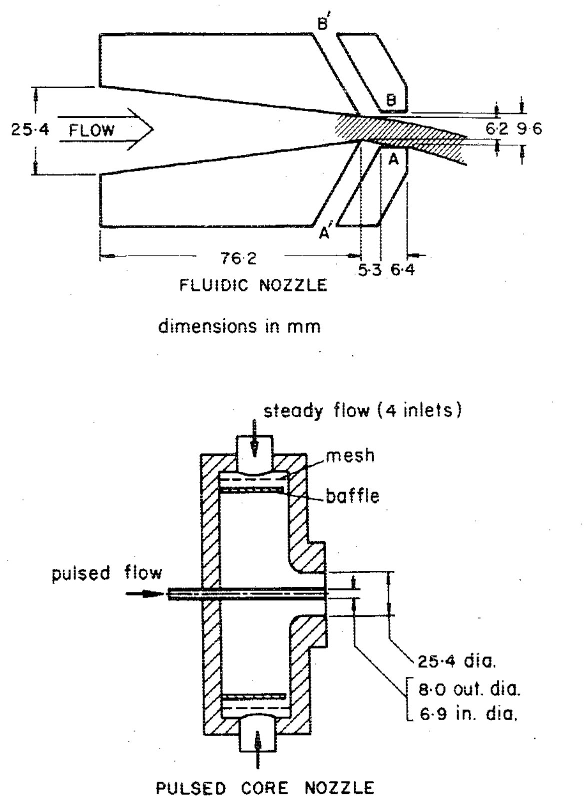

Fig. 1 Schematic diagram of fluidic and pulsed core nozzles.

Fluidically Oscillated Three-Dimensional Jet

The fluidic nozzle illustrated in Fig. 1 was used by the first and second authors to exhaust a jet of air with oscillating angle into still air. The nozzle was based on a design by Viets.[5] Flow from a plenum chamber and a contraction emerges from a 6.2×49.0 mm rectangular section into a rapid diffusion section where it is bistable because of the proximity of the walls. The flow is illustrated at the moment it attaches to the lower wall A. This sets up an entrainment process and generates compression and rarefaction waves in the feedback tube connecting control parts A' and B'. Continuous jet oscillation results at a frequency which depends on the length of the feedback tube.

In both the oscillating and the steady tests the nozzle was operated at a pressure ratio of 1.13 to produce a mean mass flow rate of 0.0188 kg/s as measured with an upstream orifice plate. The jet oscillated through about 7 deg either side of the nozzle center line and with a fundamental frequency of 52 Hz. However, higher harmonics were appreciable because of the flip-flop mode of operation. Viets[5] showed that velocity fluctuations at the half-width position of the mean velocity profile have almost a square wave shape.

The values of volumetric flow rate $Q(x)$ used in Fig. 2 were obtained by integration of the mean velocity distribution across the jet cross sections. The limits of integration were stations at which the mean velocity was between 5 and 10% of the maximum value in a distribution. This necessitated mild extrapolation of the distribution furthest downstream so that the value of $Q(x)$ there has a possible error of about 10%.

For the two cases of oscillating and fixed jet angle, $Q(x)$ is normalized by the mean volumetric flow rate Q_E at the nozzle exit. Mass flow rate upstream of the nozzle (measured with an orifice plate) was used to determine Q_E. The hydraulic diameter of the nozzle ($4 \times$area/perimeter) is used as the length scale because of the essentially three-dimensional nature of the flow. The change in slope of the curve of $Q(x)/Q_E$ for the steady jet is attributed to the transition from a high aspect ratio three-dimensional flow to a more axisymmetric mean flow.

Mechanically Oscillated Two-Dimensional Jet

Recent two-dimensional studies[6] of flow past an airfoil at zero incidence and with an oscillating trailing-edge jet flap have been extended by the first and second authors to measurements of entrainment. The nozzle was oscillated

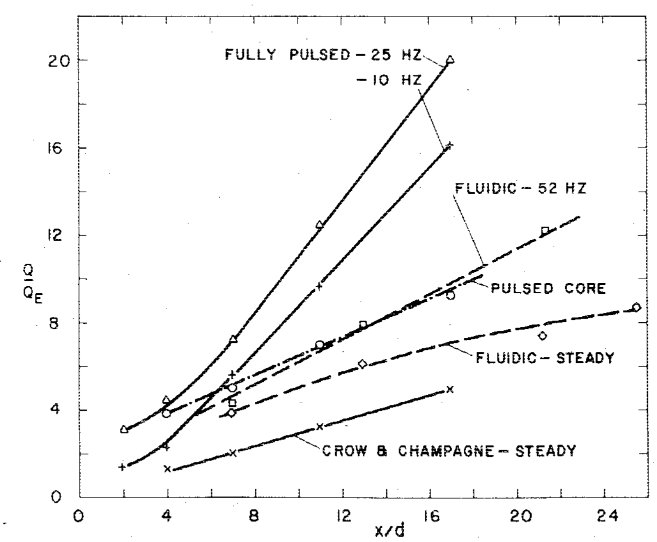

Fig. 2 Mean volumetric flow rates $Q(x)$ vs streamwise distance x for fluidic, fully pulsed and pulsed core nozzle. Q_E is mean nozzle exit flow rate and d is hydraulic diameter of fluidic nozzle or diameter of axisymmetric nozzles.

through 5.2 deg either side of the airfoil chordline and at frequencies of 4 and 20 Hz. The freestream velocity was 29.2 m/s and the nozzle exit velocity was 137 m/s. No measurable sensitivity of $Q(x)/Q_E$ to the frequency of oscillation was observed at stations between 340 and 1580 nozzle widths downstream of the nozzle exit.

Axisymmetric Jet with Pulsed Core

Bremhorst and Harch[15] recently studied a fully pulsed axisymmetric air jet exhausting into still air and their measurements of $Q(x)/Q_E$ are reproduced in Fig. 2. They used a mechanical valve connected to a plenum chamber by a smooth transition piece. The valve allowed flow for one third of its period of cyclic operation. The first and third authors used the same valve to study an axisymmetric air jet flowing into still air but with pulsation restricted to the inner core by the fitting of a two-stream coaxial nozzle downstream of the valve (Fig. 1). The nozzle consisted of a central reducer with 6.9-mm exit diameter to which air was supplied solely from the pulsating valve, and an annular section of 25.4-mm diameter, which was fed through a regulating valve with air taken from upstream of the plenum chamber.

The total jet flow rate was measured with a flow meter well upstream of the plenum chamber. The inner coaxial jet flow rate for the pulsed core was metered separately upstream of the plenum chamber. The mean exit velocities for the steady annular portion of the jet and the pulsed core were 18.3 and 12.6 m/s, respectively.

The results in Fig. 2 were obtained by planimeter integration of the radius times local mean velocity vs radius profiles. These profiles were faired to zero in order to exclude the potential tails as was done by Crow and Champagne.[14] The total volumetric flow rate across a downstream section was then normalized by the mean volumetric flow rate at the nozzle exit. Measurements by Crow and Champagne[14] for a steady axisymmetric jet are presented for comparison.

Discussion

The results in Fig. 2 show the powerful effect of full jet pulsation on entrainment. Also, the entrainment is seen to increase with frequency, but measurements are available for only two frequencies. Pulsation of only the jet core still provides significant entrainment benefits over the steady jet (Fig. 2), and this method can be regarded as an entrainment control device which enables the setting of the desired entrainment level for a jet of given flow rate. The fluidically oscillated jet shows equally significant entrainment increases [up to 55% increase in $Q(x)/Q_E$ at the most downstream station] when compared in Fig. 2 with the steady jet. Similar results with the same fluidic nozzle operated at a higher pressure ratio (1.33) were obtained by Veltman[16] with a cruder measuring technique (pitot-static tube). Finally, the volumetric flow rate measurements for the sinusoidally oscillated jet flap showed negligible variation from the corresponding steady jet measurements. This indicates that any significant influence of jet oscillation on the entrainment processes must, if it exists, be confined to the as yet uninvestigated vicinity of the nozzle.

These results indicate that entrainment depends on the type and amount of jet unsteadiness. Apparently, the mere introduction of jet unsteadiness by small sinusoidal flow angle variations is insufficient to enhance entrainment, but it should be noted that measurements were obtained at stations which are all many nozzle widths downstream of the jet nozzle. Thus, no fully conclusive statement can be made at this time about the entrainment close to the nozzle. However, the measuring stations for the sinusoidally oscillated jet were all within less than one half of the jet wavelength. Therefore, the sinusoidally oscillated jet was operated at a much smaller reduced frequency than the other two jets. In effect, it approached quasisteady conditions which may well explain its low entrainment. Indeed, in a previous paper[6] it was shown that quasisteady concepts are quite successful in explaining the major flow features.

The high entrainment of the fluidically oscillated jet would appear to be caused by the high-frequency content of this square wave type of oscillation, but more detailed measurements are clearly needed, in particular for the fluidically oscillated and the pulsed jets. Such studies are presently in progress. Furthermore, practical ejector application requires the proper tradeoff between entrainment and primary nozzle thrust efficiency. While some information is available on the thrust efficiency[5,16] of the fluidic nozzle there seems to be none available for pulsating nozzles.

Acknowledgments

The experiments reported in this paper were conducted while the first author was a Visiting Research Fellow at the University of Queensland. The authors are grateful for the support received from the Department of Mechanical Engineering, University of Queensland, The Australian Research Grants Committee under reference no. F70/17452 and the Naval Air Systems Command, U.S.A., under code AIR-310. Thanks are also due to W. Harch and M. Benson for their contributions to the experiments and to T. Smith for his assistance with the computations.

References

[1] Schum, E. F., "Techniques for Increasing Jet Entrainment Rates in Ejector Augmenters," *Proceedings of the Workshop on Prediction Methods for Jet V/STOL Propulsion Aerodynamics*, Naval Air Systems Command, July 1975, pp. 639-652, (available from DDC as AD-A024023).

[2] Quinn, B., "Compact Ejector Thrust Augmentation," *Journal of Aircraft*, Vol. 10, Aug. 1973, pp. 481-486.

[3] Chigier, N. A. and Chervinsky, A., "Experimental and Theoretical Study of Turbulent Swirling Jets Issuing from a Round Orifice," *Israel Journal of Technology*, Vol. 4, Feb. 1966, pp. 44-54.

[4] Hill, W. and Greene, P., "Self-Excited Superturbulence: The Whistler Nozzle," Grumman Research Department Rept. No. RE-488, Oct. 1974 (available from DDC as AD-A001021).

[5] Viets, H., "Flip-Flop Jet Nozzle," *AIAA Journal*, Vol. 13, Oct. 1975, pp. 1375-1379.

[6] Simmons, J. M., Platzer, M. F., and Smith, T. C., "Velocity Measurements in an Oscillating Plane Jet Issuing into a Moving Airstream," *Journal of Fluid Mechanics*, to appear, 1978.

[7] Lockwood, R. M., "Pulse Reactor Lift-Propulsion System Development Program," Hiller Aircraft Company, Rept. No. ARD-308, March 1963.

[8] Foa, J. V., "Intermittent Jets," *High-Speed Aerodynamics and Jet Propulsion*, Vol. XII, Sec. F, Princeton University Press, Princeton, N.J., 1958.

[9] Bertin, J., "Dilution Pulsatoire sur Reacteur, *Comptes Rendus, Académie des Sciences* (Paris), Vol. 240, May 1955, pp. 1855-1857.

[10] Curtet, R. M. and Girard, J. P., Visualization of a Pulsating Jet," *Proceedings of ASME Symposium on Fluid Mechanics of Mixing*, Atlanta, Ga., June 1973, pp. 173-180.

[11] Johnson, W. S. and Yang, T., "A Mathematical Model for the Prediction of the Induced Flow in a Pulsejet Ejector with Experimental Verification," *ASME Winter Annual Meeting and Energy Systems Exposition*, Paper 68-WA/FE-33, New York, Dec. 1968.

[12] Didelle, H., Binder, G., Craya, A., and Laty, R., "L'Augmentation de la Poussée d'une Trompe a Jet Inducteur Pulsant: Résultats des Essais," Laboratoires de Mécanique des Fluides, Univ. Grenoble, June 1972.

[13] Binder, G. and Favre-Marinet, M., "Mixing Improvement in Pulsating Turbulent Jets," *Proceedings of ASME Symposium on Fluid Mechanics of Mixing*, Atlanta, Ga., June 1973, pp. 167-172.

[14] Crow, S. C. and Champagne, F. H., "Orderly Structure in Jet Turbulence," *Journal of Fluid Mechanics*, Vol. 48, Aug. 1971, pp. 547-591.

[15] Bremhorst, K. and Harch, W. H., "Near Field Velocity Measurements in a Fully Pulsed Subsonic Air Jet," *Symposium on Turbulent Shear Flows*, Pennsylvania State University, April 1977.

[16] Veltman, R. J., "An Experimental Investigation of the Efficiency and Entrainment Rates of a Fluidically Oscillated Jet," Master's Thesis, U.S. Naval Postgraduate School, June 1976.